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Towards a Grammar of Bayesian Coherentism

Abstract. One of the integral parts of Bayesian coherentism is the view that the relation of 'being no less coherent than' is fully determined by the probabilistic features of the sets of propositions to be ordered. In the last one and a half decades, a variety of probabilistic measures of coherence have been put forward. However, there is large disagreement as to which of these measures best captures the pre-theoretic notion of coherence. This paper contributes to the debate on coherence measures by considering three classes of adequacy constraints. Various independence and dependence relations between the members of each class will be taken into account in order to reveal the 'grammar' of probabilistic coherence measures. Afterwards, existing proposals are examined with respect to this list of desiderata. Given that for purely mathematical reasons there can be no measure that satisfies all constraints, the grammar allows the coherentist to articulate an informed pluralist stance as regards probabilistic measures of coherence.

Keywords: Probabilistic measures of coherence, Adequacy constraints, Formal epistemology.

Introduction

One of the former leading proponents of a coherence theory of justification, Laurence BonJour, renounced his coherence theory, set out in his *The Structure of Empirical Knowledge* (1985), 14 years later complaining that "the precise nature of coherence remains a largely unsolved problem" ([6], 124). Ironically, this very same year was the starting point of a fruitful new branch of formal epistemology, viz. *Bayesian coherentism*. One of the integral components of this position is the view that the relation of 'being no less coherent than' is fully determined by the probabilistic features of the sets of propositions under consideration. Accordingly, many Bayesian coherentists subscribe to the view that one can assess the degree of coherence of a given

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set of propositions by constructing a measure that takes into account solely probabilistic information relating the propositions in the set.[1]

Initiated by Shogenji [59], many probabilistic measures of coherence have been proposed since then. Early suggestions were made by Glass [24], Olsson [44] and Fitelson [21]. Alongside their own measure, Douven and Meijs [15] put forward a recipe for coherence measures. The basic idea underlying this recipe is that coherent propositions mutually support each other and that the degree of support a proposition provides for another is best captured by means of a probabilistic measure of confirmation. Accordingly, the multitude of existing confirmation measures allows for the construction of many different coherence measures.[2] Beyond that, Schupbach [56] proposed a refined version of Shogenji's measure and a refined version of the Glass–Olsson measure is due to Meijs [41]. This plurality of coherence measures seems to be based on different and sometimes even conflicting intuitions and it is far from clear, whether it is possible to find a coherence measure mediating between these various proposals.[3]

This paper contributes to this debate by considering a list of desiderata for quantitative definitions of coherence. The formal results will be twofold: first, it will be shown that these desiderata, although reasonable, are jointly inconsistent. Hence, there can be no coherence measure that satisfies all desiderata. Therefore, a pluralist stance regarding probabilistic measures of coherence seems the only available option if we refrain from dismissing one of the desiderata (cf. [50]). Accordingly, the second contribution of the present discussion will be to provide a breakdown of reasonable desiderata that allows the coherentist to embrace a sophisticated pluralism as regards the adequacy of probabilistic measures of coherence.

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[1]Recent 'impossibility results' [7,8,45] purport to show that none of these measures is truth-conducive in the rough sense that more coherent sets of propositions are more likely to be true. However, these results are not beyond reasonable doubt. For discussions of these results and/or defenses see Angere [2,3], Duddy [16], Huemer [34,35], Meijs and Douven [42], Roche [47], Schubert [53], Schupbach [55] and Wheeler [65].

[2]For discussions of Bayesian confirmation measures see Brössel [10], Crupi et al. [13,14], Eells and Fitelson [17], Festa [18], Fitelson [19], Iranzo and Martnez de Lejarza [36] and Zalabardo [66]. For discussions of the corresponding measures of coherence see Olsson and Schubert [46], Roche [47], Schippers [50], Schubert [54] and Siebel and Wolff [63].

[3]One such account is tentatively proposed by Meijs in his dissertation *Probabilistic Measures of Coherence* ([40], p. 66f.).

1. A Constraint-Based Account to Coherence

Striving for a quantitative definition of coherence, the following desiderata seem beyond reasonable doubt. First, it goes without saying that the definition should capture what this notion is ordinarily taken to be about. Among the most basic intuitions concerning coherence are that coherent propositions 'hang together' and that coherence comes in degrees. Furthermore, coherence is symmetrical; i.e., the order of propositions to be examined should not affect the assigned degree of coherence (cf. [59]). All existing proposals concur at least with these fundamental desiderata. But of course, many other probabilistic measures that no one would seriously regard as a measure of coherence do so likewise. To illustrate, consider the following measure:

$$\mathcal{C}_\wedge(\varphi_1, \ldots, \varphi_n) = \Pr(\varphi_1 \wedge \ldots \wedge \varphi_n)$$

$\mathcal{C}_\wedge$ is symmetrical, allows for graded assignments of coherence and quantifies the probability of the set-theoretic intersection of the propositions under consideration. Therefore, in a way, the assigned number is also proportional to the degree to which the propositions 'hang together' in a more literal sense. However, considered *as a measure of coherence*, $\mathcal{C}_\wedge$ is a poor candidate (cf. [45, p. 98f])

Thus, in order to constrain the amount of reasonable coherence measures in a more promising way, the list of desiderata needs to be expanded. More precisely, we have to spell out what it means for propositions to 'hang together'. This will be done in the next section.

1.1. Dependence, Independence and Confirmation

According to Shogenji, "the crudest way of unpacking the idea that coherent beliefs 'hang together' is that they are either true together or false together" ([59], p. 338). In this sense, the truth of one proposition should have an influence on the truth of the others; i.e., coherent propositions should be more likely to be true together than propositions that are related neutrally. This neutral point, i.e. the point at which propositions are neither coherent nor incoherent, is then identified with probabilistic independence. Accordingly, Shogenji explicates the notion of 'hanging together' in terms of a deviation from probabilistic independence. For each coherence measure $\mathcal{C}$ let $\beta_\mathcal{C}$ denote its 'point of neutrality', i.e. the threshold

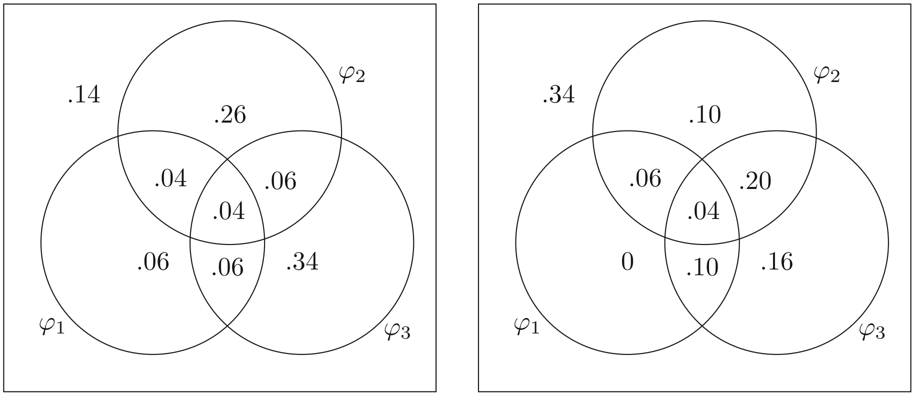

Figure 1. Diagrams of the probability distributions Pr (*left*) and Pr′ (*right*)

value that is assigned to propositions that are neither judged coherent nor incoherent.[4]

> **(CDI)** *Coherence and deviation from independence*
> Let $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ be a set of contingent propositions that are probabilistically independent for some probability function Pr, then $\mathcal{C}_{\mathrm{Pr}}(\Delta) = \beta_{\mathcal{C}}$.

According to (**CDI**), propositions that are probabilistically independent ought to be assessed neither coherent nor incoherent. In this sense, independent propositions do not hang together at all.[5] However, Fitelson [21] points to the fact that there are sets of propositions that are n-wise independent but j-wise dependent for all $j < n$. By way of illustration, consider two probability functions $\mathrm{Pr}, \mathrm{Pr}'$ over the propositional set $\{\varphi_1, \varphi_2, \varphi_3\}$ (see Fig. 1).[6] Since $\mathrm{Pr}(\varphi_1 \wedge \varphi_2 \wedge \varphi_3) = \prod_{i \leq 3} \mathrm{Pr}(\varphi_i)$ and $\mathrm{Pr}'(\varphi_1 \wedge \varphi_2 \wedge \varphi_3) = \prod_{i \leq 3} \mathrm{Pr}'(\varphi_i)$, presupposing **(CDI)** requires to assign equal degrees of coherence to $\{\varphi_1, \varphi_2, \varphi_3\}$ on both probability distributions. However, there is a decisive difference between Pr and Pr′, which is that while all propositions are also pairwise independent on Pr, no pair of propositions is independent on Pr′. Hence, these propositions are *3*-wise independent but *2*-wise

[4]Note that it is not always clear whether each coherence measures features such a threshold. One such example is the coherence measure proposed independently by Glass [24] and Olsson [44].

[5]The probabilistic coherence measure that is most directly related to (**CDI**) is due to Shogenji [59].

[6]Probability distribution Pr′ is taken from George [23].

dependent on $\Pr'$. The following notion of mutual independence specifies the difference between both situations.

Definition 1.1. $\varphi_1, \ldots, \varphi_n$ are called *mutually independent for some probability function* Pr iff for every $k = 2, 3, \ldots, n$ and every subset $\{i_1, \ldots, i_k\} \subseteq \{1, \ldots, n\}$ of distinct natural numbers $\Pr(\bigwedge_{j \leq k} \varphi_{i_j}) = \prod_{j \leq k} \Pr(\varphi_{i_j})$.

Accordingly, the propositions in a set $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ are mutually independent iff for all non-empty subsets $\Gamma \subseteq \Delta$, the probability that all propositions in Γ are true equals the product of their marginal probabilities $\Pr(\varphi_i)$ for $\varphi_i \in \Gamma$. Given the notion of mutual independence, consider the following strengthening of **(CDI)**:

> ($\mathbf{CDI}_r$) *Coherence and deviation from independence (refined version)*
> Let $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ be a set of contingent propositions that are mutually independent for some probability function Pr, then $\mathcal{C}_{\Pr}(\Delta) = \beta_{\mathcal{C}}$.

Thus, according to **(CDI$_r$)** propositions are neither coherent nor incoherent only in case of mutual independence.[7] It is easy to see that the following relationship holds between this and the former constraint[8]:

Theorem 1.2. **(CDI)** *entails* **(CDI$_r$)***, but not vice versa.*

Recall the two probability distributions given in Fig. 1. While φ_1, φ_2 and φ_3 are mutually independent on probability distribution Pr, they are pairwise *positively dependent* on $\Pr'$: for each pair of propositions φ_i, φ_j it holds that $\Pr'(\varphi_i \wedge \varphi_j) > Pr'(\varphi_i) \cdot \Pr'(\varphi_j)$, or equivalently, $\Pr'(\varphi_i \mid \varphi_j) > \Pr'(\varphi_i)$. Hence, it seems reasonably to judge the set $\{\varphi_1, \varphi_2, \varphi_3\}$ *more* coherent on $\Pr'$ than on Pr. This is also in line with BonJour's initial suggestion: the fact that elements of a given set of propositions are positively dependent indicates that inductive inferential relations obtain among its members. Hence, to yield a measure in accordance with BonJour's suggestion on the impact of inferential relations on coherence [5, p. 93], **(CDI$_r$)** has to be extended accordingly. The usual way to do so is by means of the well-known *relevance criterion* [33] according to which a proposition ψ confirms/disconfirms another proposition φ if the posterior probability of φ given ψ exceeds/falls short of its prior probability. In case of identity, ψ is neutral towards φ. The following generalization to the case of n propositions suggests itself:

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[7] For a coherence measure based on ($\mathbf{CDI}_r$) see Schupbach [56].

[8] A proof of this and all other theorems is given in the Appendix.

Definition 1.3. $\varphi_1, \ldots, \varphi_n$ are called *mutally confirmatory/disconfirmatory for some probability function* Pr if for all pairs of non-empty, non-overlapping subsets $\Sigma, \Gamma \subset \{\varphi_1, \ldots, \varphi_n\}$, $\bigwedge_{\varphi \in \Sigma} \varphi$ confirms/disconfirms $\bigwedge_{\psi \in \Gamma} \psi$.

Definition 1.3 does not take into account the relation of 'being related neutrally towards each other'. The reason for not doing so is the following:

Observation 1.4. *$\varphi_1, \ldots, \varphi_n$ are mutually independent for some probability function* Pr *iff for all pairs of non-empty, non-overlapping subsets $\Sigma, \Gamma \subset \{\varphi_1, \ldots, \varphi_n\}$, $\bigwedge_{\varphi \in \Sigma} \varphi$ is neutral towards $\bigwedge_{\psi \in \Gamma} \psi$.*

Accordingly, the following adequacy constraint expands **(CDI$_r$)** in order to account for sets of mutually dependent propositions in a satisfactory way.[9]

> **(CMS)** *Coherence and mutual support*
> Let $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ be a set of mutually confirmatory/independent/disconfirmatory propositions for some probability Pr, then $\mathcal{C}_{\mathrm{Pr}}(\Delta) > / = / < \beta_{\mathcal{C}}$.

Theorem 1.5. **(CDI)** *and* **(CMS)** *are logically independent.*

The following theorem completes the survey of logical relationships for the list of desiderata considered so far.

Theorem 1.6. **(CMS)** *entails* **(CDI$_r$)** *but not vice versa.*

Now we turn to another intuition concerning coherence that has been put forward by Bovens and Olsson [9]. Dwelling upon the relation of coherence and mutual support for pairs of propositions, they assert that "an information pair is the more coherent, the more likely each proposition becomes given the truth of the other proposition" [9, p. 688]. This idea matches another concept of confirmation which is called *absolute confirmation* or confirmation as *firmness* (cf. [11,48]). According to this concept, a proposition ψ confirms another proposition φ if the posterior probability of φ given ψ exceeds some threshold θ for $0.5 \leq \theta < 1$. Furthermore, BonJour's requirement of 'probabilistic consistency' bears some resemblance with (**CMS**) as is revealed by the following passage on the importance of "various kinds of probabilistic connections" ([6], p. 124). There he states the following:

> One important aspect of this is what might be called probabilistic consistency, i.e. the minimizing of relations between beliefs in the system

[9]For coherence measures in line with (**CMS**) see Fitelson [21,22] and Douven and Meijs [15].

in virtue of which some are highly unlikely to be true in relation to others.

Generalizing Bovens and Olsson's constraint to n-membered sets yields the following constraint on the impact of the relation of degrees of varying firmness on coherence[10]:

(**CMF**) *Coherence and mutual firmness*
Let Δ be a set of propositions and $\mathrm{Pr}, \mathrm{Pr}'$ two probability functions such that for all pairs of non-empty, non-overlapping subsets Σ, Γ of Δ, $\mathrm{Pr}\left(\bigwedge_{A\in\Sigma} A \mid \bigwedge_{B\in\Gamma} B\right) \gtreqless \mathrm{Pr}'\left(\bigwedge_{A\in\Sigma} A \mid \bigwedge_{B\in\Gamma} B\right)$. Then $\mathcal{C}_{\mathrm{Pr}}(\Delta) \gtreqless \mathcal{C}_{\mathrm{Pr}'}(\Delta)$.

Thus, according to **(CMF)** it is only by means of posterior probabilities that degrees of coherence are kept apart. For example, assuming that these posteriors are identical on two different probability functions, the assigned degrees of coherence should be identical likewise. This may even be so if the propositions are positively relevant on one distribution and negatively relevant on the other.[11] Accordingly, it is not surprising that (**CMF**) is inconsistent with all other constraints considered so far.

Theorem 1.7. **(CMF)** *is inconsistent with each of* **(CMS)**, **(CDI)** *and* **(CDI$_r$)**.

Figure 2 is a graphical representation of the logical relationships considered so far.

1.2. Logical Equivalence, Striking Agreement and Inconsistency

According to BonJour [5], the coherence of a set of propositions is increased by the presence of inferential relations among its members and increased "in proportion to the number and strength of such connections" (p. 98). Among these are "inferential, evidential and explanatory relations" (p. 93). Thus, deductive entailment, representing the prototype of an inferential relation,

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[10]Bovens and Olsson hint at another generalization that only takes into account the degrees of firmness of each subset on the remainder of the given set. Schippers [49] presents an argument that shows the limits of this account and, accordingly, argues for the superiority of (**CMF**).

[11]For an argument to the effect that sets of negatively relevant propositions may nonetheless exhibit a high degree of coherence see Siebel [61].

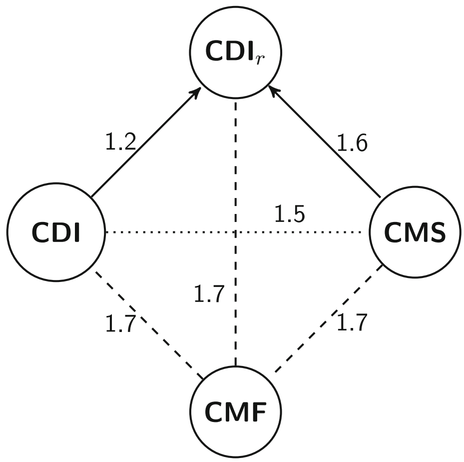

Figure 2. Logical relationships among independence and dependence conditions. *Arrows* represent entailment relations; *dotted lines* denote relationships of logical independence; *dashed lines* indicate logical inconsistency

is only one among several coherence-boosting relations. Evidential relations, on the other hand, are often equated with *inductive* inferential relations.[12]

According to a long-standing traditional view, inductive logic is considered a *generalization* of classical deductive logic [11]. A more recent repercussion of Carnap's position is to be found in Fitelson's landmark, *Studies in Bayesian Confirmation Theory* [20]. In search of a probabilistic explication of the notion of evidential support, Fitelson (p. 42) demands that in case a piece of evidence E deductively entails a hypothesis H,[13]

> the strength of the support E provides for H should *not* depend on how probable H is (*a priori*). [...] After all, evidential support is supposed to be a measure of how strong the evidential *relationship* between E and H is, and deductive entailment is the strongest that such a relationship can possibly get. If E is *conclusive* for H, then H's *a priori* probability should, intuitively, be *irrelevant* to how strong the (maximal, deductive) evidential relationship between E and H is.

[12]Thus, in his entry on "*inductive logic*" in the *Stanford Encyclopedia of Philosophy*, Hawthorne [28] defines an inductive logic as "a system of evidential support that extends deductive logic to less-than-certain inferences".

[13]A critical remark on the overall tenability of this constraint is given by Siebel and Wolff (2008, p. 176). However, as I do *not* want to endorse this as a constraint for measures of evidential support per se, I set these and other related issues.

Accordingly, *mutual* entailment is often considered a case of *maximal coherence* [7, p. 32, 21, Glass [25], 63].

(**CLE**) *Coherence and logical equivalence*
Let $\Sigma = \{\varphi_1, \ldots, \varphi_n\}$ be a set of logically equivalent propositions, than $\mathcal{C}_{\mathrm{Pr}}(\Sigma)$ is maximal for all probability functions Pr.

To illustrate, imagine a fair die is rolled and consider the two possible outcomes

(A_1) 'The die will come up 2.'

(A_2) 'The die will come up an even number less than 4.'

(A_1) and (A_2) are logically equivalent and seem to fit together perfectly. Thus, it seems reasonable to require a maximum degree of coherence.[14] As opposed to equivalent propositions being maximally coherent, many scholars agree in that inconsistency has a negative impact on a set's degree of coherence.[15] In what follows we distinguish two notions of inconsistency for sets of propositions, viz. weak inconsistency and strong inconsistency.

Definition 1.8. Let $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ be an inconsistent set of contingent propositions. Δ is *strongly inconsistent* iff all subsets $\Gamma \subseteq \Delta$, $|\Gamma| \geq 2$ are unsatisfiable. Otherwise Δ is called *weakly inconsistent*.

It goes without saying that no pair of propositions can be weakly inconsistent. On the other hand, the set $\{\varphi_1, \varphi_1 \rightarrow \varphi_2, \neg\varphi_2\}$ is weakly inconsistent: although the set itself is inconsistent, there are consistent subsets like $\{\varphi_1, \varphi_1 \rightarrow \varphi_2\}$ and $\{\varphi_1 \rightarrow \varphi_2, \neg\varphi_2\}$. Given these two different qualitative notions of "degrees" of inconsistency, we can reasonably keep apart the following two desiderata: on the one hand, presumably all scholars would agree that a strongly inconsistent set is maximally incoherent. This is the *weak* constraint on coherence measures.

($\mathbf{CLI}_w$) *Coherence and logical inconsistency – weak version*
Let $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ be strongly inconsistent, then $\mathcal{C}_{\mathrm{Pr}}(\Delta) = \min$ for every probability function Pr.

This constraint leaves room for different degrees of incoherence for weakly inconsistent sets. On the other hand, one might also be inclined to judge *all*

[14] Nevertheless, **(CLE)** is *not* beyond reproach (cf. [15, 41, 42, 46]).

[15] cf. BonJour [5, p. 95], Fitelson [21] and Roche [47].

inconsistent sets (either weakly or strongly inconsistent) are maximally incoherent.[16] One rationale for doing so might be the following: it is indeed the case that inferential relations are among the key coherence boosting factors [5, p. 93], but these relations often abound in inconsistent sets due to the fact that classical logic is *explosive*: it validates the inference from $\langle\varphi, \neg\varphi\rangle$ to ψ for every pair of propositions φ, ψ. Consequently, it seems that in the presence of inconsistencies, the property of *explosion* causes a loss of sensitivity for genuine coherence-boosting inferential relations as compared to mere "degenerated" entailment relations. Accordingly, we get the following desideratum:

> (**CLI**$_\mathbf{s}$) *Coherence and logical inconsistency – strong version*
> Let $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ be weakly inconsistent, then $\mathcal{C}(\Delta) = \min$ for every probability function Pr.

The following relationship is easily seen to obtain between the two desiderata linking inconsistency and (in)coherence.

Theorem 1.9. *(CLI$_s$) entails (CLI$_w$), but not vice versa.*

The requirement that equivalent propositions be maximally coherent naturally follows from a view of coherence as a probabilistic generalization of logical equivalence (see [21,41]). On this view, logical equivalence and inconsistency should be the sharp edges of maximal and minimal coherence. However, Meijs [41] argues that the combination of (**CLE**) and (**CMS**) leads to counter-intuitive results when applied to examples involving sub-contrary propositions. Further congenial scholars who disagree with the maximality-requirement are Olsson [45] and Schubert [54]. What unites them is the opinion that, intuitively, the degree of coherence of a set of equivalent testimonies is proportional to its informativity: while equivalent, but nearly tautologous testimonies exhibit only a very low degree of coherence, highly specific equivalent propositions are also highly coherent. In other words, the degree of coherence of equivalent propositions depends on how striking this agreement is.

According to a common view, the amount of information conveyed by a proposition is inversely related to its probability. For example, the information that a fair die did not came up 6 is much less specific than that it came up 2. Similarly, the chance that it did not come up six is much higher than

[16] A position along these lines seems to be endorsed by Bovens and Hartmann [8, p. 608]. Furthermore, given Shogenji's formal reconstruction of coherence, he is also committed to this stronger inconsistency requirement. However, see also Shogenji [60].

the corresponding chance that it came up 2. Two classical representations of the degree of information conveyed by φ are the following[17]:

(i) $inf_R(\varphi) = \log_2[1/\Pr(\varphi)]$

(ii) $inf_D(\varphi) = 1 - \Pr(\varphi)$

In what follows, I use inf (without subscript) in order to denote either inf_R (the *ratio* based measure) or inf_D (the *difference* based measure).[18] Both measures satisfy the following conditions:

(i) $inf(\top) \leq inf(\varphi) \leq inf(\bot)$ for all contingent φ.

(ii) If $\varphi \vDash \varphi'$, then $inf(\varphi') \leq inf(\varphi)$ for all φ, φ'.

Given a regular probability function, according to (i) a tautology is assigned the minimum degree of information while a contradiction $\bot$ is assigned the maximum degree of information. Furthermore, information content varies with logical strength (ii). Based on these information measures, we can formulate the opposite requirement for equivalent testimonies as follows:

> (**CSE**) *Coherence and the strike of equivalence*
> Let $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ be a set of logically equivalent propositions, than $\mathcal{C}_{\Pr}(\Delta)$ is proportional to $inf(\varphi_i)$ for some $1 \leq i \leq n$.

According to (**CSE**), the degree of coherence of logically equivalent propositions is inversely proportional to the propositions' prior probability. Hence, it should not be surprising that (**CSE**) and (**CLE**) are inconsistent.

THEOREM 1.10. *(**CLE**) and (**CSE**) are logically inconsistent.*

Both these constraints, however, are compatible with ($\mathbf{CLI}_w$) and ($\mathbf{CLI}_s$) but also with their negations. That is, those pairs of constraints are logically independent.

THEOREM 1.11. *(**CLE**) and ($\mathbf{CLI}_i$) are logically independent for $i \in \{w, s\}$.*

THEOREM 1.12. *(**CSE**) and ($\mathbf{CLI}_i$) are logically independent for $i \in \{w, s\}$.*

Figure 3 is a graphical representation of the logical relationships between the constraints considered within this chapter.

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[17] These measures have been discussed in seminal work by Bar-Hillel and Carnap [4].

[18] inf_R has also prominent applications in so-called information theory as founded by Shannon [57]. See Shannon and Weaver [58]. For further discussion of these measures see Hintikka [29,30] and Hintikka and Pietarinen [31].

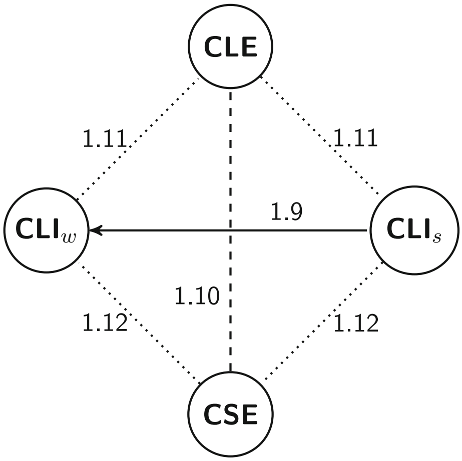

Figure 3. Logical relationships among constraints regarding equivalence and inconsistency. *Arrows* represent entailment relations; *dotted lines* denote relationships of logical independence; *dashed lines* indicate logical inconsistency

1.3. Logical Entailment and Disagreement

Assume that Anne and Bob disagree with respect to the color of Catherine's hair. For example, stipulate that Anne thinks Catherine's hair is blond while Bob thinks it is not. This is a case of contradictory beliefs and it seems that Anne's and Bob's beliefs are highly, if not maximally, incoherent. On the other hand, we can also alleviate the assumptions regarding their disagreement in various ways. First, it could be the case that Anne still believes that she has blond hair while Bob believes that Catherine's hair is brown. These beliefs are no longer contradictory but only contrary. Nonetheless, they still seems to be incoherent. Equivalently, if Anne believes that Catherine's hair is not blond while Bob believes it is not brown, their beliefs are subcontrary. However, they still seem incoherent to some degree.

On the other hand, their disagreement might relate to possibilities that are admitted by Anne but not by Bob and vice versa. Even if Bob's belief is logically entailed by Anne's, partial disagreement with respect to possible colors of Catherine's hair might persist. To illustrate, assume that Anne believes that Catharine's hair is colored blond while Bob thinks it is either blond or brown. It seems, however, that this sort of disagreement is no reason to conclude that pairs of beliefs such that one is entailed by the other are incoherent. Quite the contrary, it seems that all those pairs of beliefs are indeed coherent.

These considerations yield a number of adequacy constraints that will be scrutinized in the current section. For means of exposition, the discussion will be based exclusively on pairs of propositions. We start the discussion by cases of entailment between pairs of propositions.

(**Ent**) If φ logically entails ψ, then φ and ψ are coherent.

According to (**Ent**), a pair of propositions such that one of them entails the other should always be assessed coherent. The intuition that the presence of inferential relations has a positive impact on the coherence of a set of beliefs is widely shared [5,27,64]. On the other hand, as was mentioned before, even for pairs of propositions where one entails the other there is room for disagreement. To illustrate, consider a court case with 10 equiprobable suspects and two witnesses Anne and Bob. Now Anne testifies that it was suspect 1 while Bob testifies it was either 1 or 2 or ... or 9. Obviously, if Anne's testimony is true then so is Bob's; however, both testimonies largely disagree with respect to the number of potential suspects. More precisely, the argument from the truth of Bob's testimony to Anne's is not only deductively invalid but also inductively weak in the sense that the conclusion is assigned a low conditional probability given the premise. Accordingly, we might strengthen the entailment desideratum (**Ent**) in various ways. One such possibility is to require additionally a high degree of conditional probability. Given the vagueness of this requirement, I prefer the following comparative constraint:

(**Ent**$_c$) If φ logically entails both ψ and ψ' and $\Pr(\varphi|\psi) > \Pr(\varphi|\psi')$ for some probability function Pr, then $\mathcal{C}_{\Pr}(\varphi, \psi) > \mathcal{C}_{\Pr}(\varphi, \psi')$.

This constraint captures the intuition that the remaining disagreement in the court case considered above is rather high. In probabilistic terms, this means that the probability that Bob's testimony is true (b) *and* Anne's testimony is false ($\neg a$), $\Pr(\neg a \wedge b)$, is high. Examining the logical relationship between those two constraints on logical entailment and coherence, one can see that it is neither the case that the qualitative coherence constraint entails the comparative constraint nor the other way round.

Theorem 1.13. *(*__Ent__*) and (*__Ent__$_c$*) are logically independent.*

Another formal rendition of this latter comparative coherence intuition is to compute the degree of disagreement for pairs of propositions and then to require the degree of coherence to be inversely related to this degree of disagreement. The following formula suggests itself as a compact representation of this degree of disagreement, $dis(\varphi, \psi)$, between φ and ψ:

$$dis(\varphi, \psi) := \Pr(\varphi \Delta \psi) = \Pr(\varphi \wedge \neg\psi) + \Pr(\neg\varphi \wedge \psi)$$

With this formula at hand, we get the following alternative constraint:

> ($\mathbf{Dis}_{\vDash}$) If φ logically entails both ψ and ψ', and $dis(\varphi,\psi) < dis(\varphi,\psi')$ for some probability function Pr, then $Coh_{\mathrm{Pr}}(\varphi,\psi) > Coh_{\mathrm{Pr}}(\varphi,\psi')$.

It turns out that there is a close relationship between these two latter constraints, viz.,

THEOREM 1.14. *($\mathbf{Ent}_c$) and ($\mathbf{Dis}_{\vDash}$) are logically equivalent.*

It is also possible to strengthen the requirement of deductive entailment that forms the common core of the constraints considered so far in this section. This is done by means of the notion of *compatibility.* The part on which two propositions agree equals the conjunction of both propositions. Now if $\varphi \vDash \psi \wedge \psi'$, as stipulated by ($\mathbf{Dis}_{\vDash}$), then $\mathrm{Pr}(\varphi \wedge \psi) = \mathrm{Pr}(\varphi \wedge \psi')$. However, it seems that there is a strengthened version of this requirement that is still in the spirit of ($\mathbf{Dis}_{\vDash}$): denote the *degree* of agreement or *compatibility* between φ and ψ by $comp(\varphi,\psi)$, i.e. $comp(\varphi,\psi) := \mathrm{Pr}(\varphi \wedge \psi)$, then it seems reasonable to require that the higher the agreement and the lower the disagreement between pairs of propositions, the higher their degree of coherence, ceteris paribus. This latter intuition is spelled out by the following constraint:

> (**Dis**) If $comp(\varphi,\psi) \geq comp(\varphi',\psi')$ and $dis(\varphi,\psi) \leq dis(\varphi',\psi')$ (and one inequality is strict), then $Coh(\varphi,\psi) > Coh(\varphi',\psi')$.

This constraint is obviously logically stronger than the former, i.e.

THEOREM 1.15. *(**Dis**) entails ($\mathbf{Dis}_{\vDash}$), but not vice versa.*

The following theorem completes the survey of disagreement-constraints.

THEOREM 1.16. *(**Ent**) and (**Dis**) are logically inconsistent.*

We finish this survey of constraints again with a graphical representation (Fig. 4). Note that although theorems 1.13 and 1.15 disregard constraint ($\mathbf{Dis}_{\vDash}$), both are true also if we replace ($\mathbf{Ent}_c$) by ($\mathbf{Dis}_{\vDash}$) due to theorem 1.14.

2. A Survey of Probabilistic Coherence Measures

This section briefly reviews existing proposals to measuring coherence. In a second step, we present a survey that specifies for each measure which of the considered constraints are satisfied and which are violated. Let $\Delta = \{\varphi_1,\ldots,\varphi_n\}$ be a set of contingent propositions, then the following is a list of prominent measures:

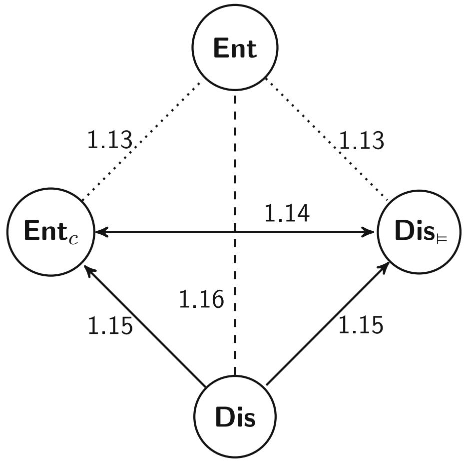

Figure 4. Logical relationships among constraints regarding logical entailment and disagreement. *Arrows* represent entailment relations; *dotted lines* denote relationships of logical independence; *dashed lines* indicate logical inconsistency

Deviation measures The first approach to explicate coherence probabilistically is based on the idea of coherence as a deviation from probabilistic independence. The most prominent measure within this class reads as follows[19]:

$$\mathcal{D}(\Delta) = \frac{\Pr(\bigwedge_{i\leq n}\varphi_i)}{\prod_{i\leq n}\Pr(\varphi_i)}$$

One of the several criticisms that this measure has faced is its purported lack of sensitivity to the coherence of subsets of a given set under consideration (cf. [56]).[20] Accordingly, Schupbach proposed a refined version of this measure that takes into account these degrees of coherence of the various subsets of a given set. In order to have a concise formal representation let $[\Delta]^k$ denote the set of all subsets of Δ with cardinality k, then the cardinality of this latter set $[\Delta]^k$ is $m_k = \binom{n}{k} = n!/k!(n-k)!$.

[19]Equivalently, one could take the difference between the probability of the conjunction and the product of the marginal probabilities. This latter measure has been considered by Carnap [11] as a measure of the degree of mutual confirmation between the propositions within a given set (cf. [39]).

[20]For other criticisms see Akiba [1], Bovens and Hartmann [7], Fitelson [21], Glass [25], Siebel and Wolff [63], Schippers [49].

$$\mathcal{D}^*(\Delta) = \frac{1}{n-1}\sum_{k=2}^{n}\sum_{\Delta_i\in[\Delta]^k}\frac{\log_{10}\mathcal{D}(\Delta_i)}{m_k}$$

Overlap measures A second idea for how to spell out the concept of coherence is based on the relative set theoretic overlap. The higher this overlap, i.e. the higher the probability that all of a set's propositions are true given that at least one is, the higher the degree of coherence. The most prominent overlap measure reads as follows:

$$\mathcal{O}(\Delta) = \frac{\Pr(\bigwedge_{i\leq n}\varphi_i)}{\Pr(\bigvee_{i\leq n}\varphi_i)}$$

Again, there is a refined version that takes into account the overlap of subsets of the set under consideration. Let $m = \sum_{2\leq k\leq n} m_k$, then Meijs' [41] overlap measure reads as follows:

$$\mathcal{O}^*(\Delta) = \frac{1}{m}\sum_{k=2}^{n}\sum_{\Delta_i\in[\Delta]^k}\mathcal{O}(\Delta_i)$$

Mutual support-based measures In what follows we sketch a recipe for coherence measures that has been proposed by Douven and Meijs [15]. The general setup can be considered a quantitative explication of BonJour's characterization of the coherence of a set as being "increased by the presence of inferential connections between its component beliefs and increased in proportion to the number and strength of such connections" (1985, p. 98). Now in order to quantify the "strength" of inferential connections, Douven and Meijs resort to the well-known debate within philosophy of science on probabilistic measures of confirmation, a.k.a. support. Let $\mathfrak{s}$ be a probabilistic measure of confirmation, i.e. a function assigning triples of two propositions φ, ψ and a probability distribution Pr a real number that is supposed to represent the degree to which ψ confirms φ. Table 1 (p. 17) lists some of the most prominent confirmation measures. The vast majority of these measures are based on the concept of *incremental confirmation* according to which ψ confirms φ if and only if $\Pr(\varphi|\psi) > \Pr(\varphi)$. Only measure f highlights the alternative confirmation concept of absolute confirmation that requires for ψ to confirm φ that $\Pr(\varphi|\psi)$ exceeds a threshold θ where $0.5 \leq \theta < 1$.[21]

[21] A discussion of corresponding coherence measure $\mathcal{C}_f$ is given by Roche [47] and Schippers [49,50].

Table 1. A survey of prominent probabilistic measures of confirmation

Measure	Definition	Advocate
$d(\varphi, \psi)$	$\Pr(\varphi\|\psi) - \Pr(\varphi)$	Carnap [11]
$r(\varphi, \psi)$	$\Pr(\varphi\|\psi)/\Pr(\varphi)$	Keynes [38]
$n(\varphi, \psi)$	$\Pr(\psi\|\varphi) - \Pr(\psi\|\neg\varphi)$	Nozick [43]
$s(\varphi, \psi)$	$\Pr(\varphi\|\psi) - \Pr(\varphi\|\neg\psi)$	Christensen [12]
$l(\varphi, \psi)$	$\Pr(\psi\|\varphi)/\Pr(\psi\|\neg\varphi)$	Good [26]
$k(\varphi, \psi)$	$n(\varphi, \psi)/[\Pr(\psi\|\varphi) + \Pr(\psi\|\neg\varphi)]$	Kemeny and Oppenheim [37]
$z(\varphi, \psi)$	$\begin{cases} d(\varphi, \psi)/[1 - \Pr(\varphi)], \text{if} \Pr(\varphi\|\psi) \geq \Pr(\varphi) \\ d(\varphi, \psi)/[\Pr(\varphi)], \text{otherwise} \end{cases}$	Crupi et al. [13]
$f(\varphi, \psi)$	$\Pr(\varphi\|\psi)$	Carnap [11]

In a nutshell, the recipe is now based on the following appealing idea: a set's degree of coherence depends on the degree of probabilistic confirmation its elements provide for each other. To quantify this degree of mutual confirmation, calculate the extent to which each proposition and conjunction of propositions is supported by each remaining proposition and conjunction of them. More formally, let $[\Delta]$ be the set of all pairs of non-empty, non-overlapping subsets of Δ, then the cardinality of $[\Delta]$ is given by $l = \sum_{1 \leq i \leq n} \binom{n}{i} \left(2^{n-i} - 1\right)$. The corresponding recipe now reads as follows:

$$\mathcal{C}_{\mathfrak{s}}(\Delta) = \sum_{(\Delta', \Delta'') \in [\Delta]} l^{-1} \cdot \mathfrak{s}(\Delta', \Delta'')$$

The *grammar* of each considered measure is specified in Table 2 (p. 18).As the table reveals, some of the measures exhibit a considerably different grammar. Based on this finding, one might either argue for the superiority of some of the considered constraints and opt for some kind of monism about measuring coherence; or one might consider this result to be evidence for a pluralist stance with respect to measuring coherence (cf. [50]). However, even if one is sympathetic to the idea of monism about measuring coherence, the incompatibilities show that there will not be a single measure that satisfies *all* considered constraints. Nonetheless, it is possible to improve the performance for some of the considered measures with respect to the inconsistency constraint ($\mathbf{CLI}_w$). Here, the measures' results are often not due to their underlying "grammatical structure" but solely caused by the fact that some of the involved probabilities are simply not defined. Accordingly, it is possible to improve the measures' performance regarding the assessment

Table 2. The “grammar” of coherence measures. + Indicates that a measure satisfies a constraint; − indicates a violation of the constraint. Cases marked with ‘?’ are so far unsettled. All proofs are given in the Appendix

	$\mathcal{D}$	$\mathcal{O}$	$\mathcal{D}^*$	$\mathcal{O}^*$	$\mathcal{C}_d$	$\mathcal{C}_r$	$\mathcal{C}_s$	$\mathcal{C}_n$	$\mathcal{C}_l$	$\mathcal{C}_k$	$\mathcal{C}_z$	$\mathcal{C}_f$
(**CDI**)	+	−	−	−	−	−	−	−	−	−	−	−
(**CDI**$_r$)	+	−	+	−	+	+	+	+	+	+	+	−
(**CMS**)	+	−	+	−	+	+	+	+	+	+	+	−
(**CMF**)	−	?	−	?	−	−	−	−	−	−	−	+
(**CLE**)	−	+	−	+	−	−	+	+	−	+	+	+
(**CLI**$_w$)	+	+	+	+	−	−	−	−	−	−	−	−
(**CLI**$_s$)	+	+	+	−	−	−	−	−	−	−	−	−
(**CSE**)	+	−	+	−	+	+	−	−	−	−	−	−
(**Ent**)	+	−	+	−	+	+	+	+	+	+	+	+
(**Ent**$_c$)	+	+	+	+	+	+	+	+	−	+	+	+
(**Dis**$_\models$)	+	+	+	+	+	+	+	+	−	+	+	+
(**Dis**)	−	+	−	+	−	−	−	−	−	−	−	−

of inconsistent sets of propositions by adapting the underlying confirmation measure.[22]

3. Conclusion

This paper presents a survey on recent work on probabilistic models of coherence. The main contribution is a sketch of constraints that allows to spell out the *grammar* of existing coherence measures in some detail. What is more, the survey of logical relationships between families of constraints might be considered an argument for a (modest) pluralism about probabilistic measures of coherence by revealing incompatibilities among the desired properties of such measures.

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[22] A survey of different approaches for adapting confirmation measures in order to assess the degree of incoherence of inconsistent sets of propositions is given by Schippers and Siebel [52]. Additionally, the question is addressed whether existing coherence measures are able to assign *different* degrees of incoherence to inconsistent sets of propositions. For an assessment of the relationship between degrees of incoherence and degrees of inconsistency see Schippers [51].

Appendix

A. Proof of Theorem 1.2

THEOREM 1.2 (**CDI**) *entails* (**CDI**$_r$)*. but not vice versa.*

PROOF. Let $\mathcal{C}$ be a measure violating (CDI$_r$). Then there is a set $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ of mutually independent propositions for some probability function Pr such that $\mathcal{C}_{\Pr}(\Delta) \neq \beta$. Since mutually independent propositions are also n-wise independent, $\mathcal{C}$ violates (**CDI**) as well. That (**CDI**$_r$) does not entail (**CDI**) is entailed by the results of Table 2: $\mathcal{D}^*$ is for example one of those measures that satisfy (**CDI**$_r$) while violating (**CDI**).

B. Proof of Theorem 1.5

THEOREM 1.5 (**CDI**) *and* (**CMS**) *are logically independent.*

PROOF. As is well known, logical independence amounts to the following two aspects:

(i) *Consistency* As for consistency, we show that Shogenji's measure satisfies both desiderata:

$$\mathcal{D}(\varphi_1, \ldots, \varphi_n) = \frac{\Pr(\bigwedge_{i \leq n} \varphi_i)}{\prod_{i \leq n} \Pr(\varphi_i)}$$

PROOF. Utilizing the chain rule, we get

$$\Pr(\varphi_1 \wedge \ldots \wedge \varphi_n) = \Pr(\varphi_1) \cdot \Pr(\varphi_2 \mid \varphi_1) \cdot \ldots \cdot \Pr(\varphi_n \mid \varphi_1 \wedge \ldots \wedge \varphi_n)$$

Now assuming that $\varphi_1, \ldots, \varphi_n$ are mutually confirmatory / independent / disconfirmatory, we get $\Pr(\varphi_1 \wedge \ldots \wedge \varphi_n) > / = / < \Pr(\varphi_1) \cdot \ldots \cdot \Pr(\varphi_n)$. Accordingly, with $\beta = 1$ measure $\mathcal{D}$ satisfies (**CMS**). Furthermore, it goes without saying that $\mathcal{D}$ also satisfies (**CDI**). Consequently, (**CDI**) and (**CMS**) are consistent desiderata.

(ii) *Non-redundancy* Consider the following two measures:

(a) $\mathcal{C}_2(\varphi_1, \ldots, \varphi_n) = \sin\Big(\pi \cdot \Pr(\bigwedge_{i \leq n} \varphi_i) \,/\, \prod_{i \leq n} \Pr(\varphi_i)\Big)$

(b) $\mathcal{C}_3(\varphi_1, \ldots, \varphi_n) = \big((n-2)! \cdot 2! \,/\, n!\big) \cdot \sum_{1 \leq i \neq j \leq n} \frac{\Pr(\varphi_i \wedge \varphi_j)}{\Pr(\varphi_i) \cdot \Pr(\varphi_j)}$

In what follows we show (a) that $\mathcal{C}_2$ satisfies (**CDI**) but violates (**CMS**), while (b) $\mathcal{C}_3$ violates (**CDI**) but satisfies (**CMS**).[23]

(a) $\mathcal{C}_2$'s range is [−1,1]. Given n-wise independent propositions $\varphi_1, \ldots, \varphi_n$, we get $\mathcal{C}_2(\varphi_1, \ldots, \varphi_n) = 0$. Thus, it seems reasonable to stipulate that 0 is the threshold separating coherent from incoherent sets. But, of course, $\mathcal{C}_2$ violates (**CMS**). To see this, consider the following probability distributions over $\Delta = \{\varphi_1, \varphi_2\}$:

φ_1	φ_2	Pr_1	Pr_2
T	T	3/32	5/32
T	F	5/32	3/32
F	T	5/32	3/32
F	F	19/32	21/32

Straightforward calculations yield the following orderings:

(i) $\mathrm{Pr}_1(\varphi_i \mid \varphi_j) = \frac{3}{8} > \frac{1}{4} = \mathrm{Pr}_1(\varphi_i)$ for $1 \leq i \neq j \leq 2$
(ii) $\mathrm{Pr}_2(\varphi_i \mid \varphi_j) = \frac{5}{8} > \frac{1}{4} = \mathrm{Pr}_2(\varphi_i)$ for $1 \leq i \neq j \leq 2$

So φ_1 and φ_2 are mutually confirmatory for both probability functions. Nonetheless, $\mathcal{C}_2^{\mathrm{Pr}_1}(\varphi_1, \varphi_2) = -1$ and $\mathcal{C}_2^{\mathrm{Pr}_2}(\varphi_1, \varphi_2) = 1$. Thus $\mathcal{C}_2$ clearly violates (**CMS**).

(b) $\mathcal{C}_3$'s neutral value is 1. Given mutually confirmatory/independent/ disconfirmatory propositions $\varphi_1, \ldots, \varphi_n$ we get $\mathcal{C}_3(\varphi_1, \ldots, \varphi_n) > / = / < 1$. So $\mathcal{C}_3$ satisfies (**CMS**). In order to show that $\mathcal{C}_3$ violates (**CDI**) recall the probability distribution Pr given in Fig. 1 on the right-hand side. Even though $\varphi_1, \varphi_2, \varphi_3$ are 3-wise independent, all *pairs* of propositions were positively dependent. Thus, in conflict with (**CDI**), $\mathcal{C}_3(\varphi_1, \varphi_2, \varphi_3) > 1$.

C. Proof of Theorem 1.6

THEOREM 1.6 (**CMS**) *entails* ($\mathbf{CDI}_r$), *but not vice versa.*

PROOF. Let $\mathcal{C}$ be a measure violating ($\mathbf{CDI}_r$). Then there is a set of mutually independent propositions $\Delta = \{\varphi_1, \ldots, \varphi_n\}$ for some probability function such that $\mathcal{C}(\Delta) \neq \beta$. Hence, $\mathcal{C}$ violates (**CMS**) alike. To see that ($\mathbf{CDI}_r$) on the other hand does not entail (**CMS**) recall measure $\mathcal{C}_2$ from the proof of

[23] Note that $\mathcal{C}_3$ is a coherence measure that only takes into account pairwise deviation from independence. Given a set of n propositions, the number of pairs equals $n! \,/\, (n-2)!$. Hence, $\mathcal{C}_3$ is the straight average of the amount of the deviation from independence for all pairs of propositions.

Theorem 1.5. This measure clearly satisfies (**CDI**$_r$) while violating (**CMS**). Accordingly, (**CDI**$_r$) does not entail (**CMS**).

D. Proof of Theorem 1.7

THEOREM 1.7 (**CMF**) *is inconsistent with each of* (**CMS**), (**CDI**) *and* (**CDI**$_r$).

PROOF. Consider the probability distributions $\Pr_3$ and $\Pr_4$ over φ_1 and φ_2. Straightforward calculations yield the following results:

φ_1	φ_2	$\Pr_3$	$\Pr_4$	$\Pr_5$
T	T	1/8	1/4	1/16
T	F	1/8	1/4	3/16
F	T	1/8	1/4	3/16
F	F	5/8	1/4	9/16

(i) $\Pr_3(\varphi_i|\varphi_j) = 1/2 > 1/4 = \Pr_3(\varphi_i)$ for $1 \leq i \leq 2$

(ii) $\Pr_4(\varphi_i|\varphi_j) = 1/2 = \Pr_4(\varphi_i)$ for $1 \leq i \leq 2$

Hence, according to (**CMS**), $\mathcal{C}_{\Pr_3}(\varphi, \psi) > \mathcal{C}_{\Pr_4}(\varphi, \psi)$, while according to (**CMF**) $\mathcal{C}_{\Pr_3}(\varphi, \psi) = \mathcal{C}_{\Pr_4}(\varphi, \psi)$. Contradiction. Thus, $\mathcal{C}$ violates (**CMS**). To see that (**CMF**) and (**CDI**) are inconsistent, consider probability distributions $\Pr_4$ and $\Pr_5$:

(iii) $\Pr_5(\varphi_i|\varphi_j) = 1/4 = \Pr_5(\varphi_i)$ for $1 \leq i \leq 2$

Accordingly, φ_1 and φ_2 are probabilistically independent on both probability distributions, $\Pr_4$ and $\Pr_5$. Therefore, each measure $\mathcal{C}$ satisfying (**CDI**) will assign the neutral value β to φ_1 and φ_2 on both distributions. On the other hand, since the posterior probabilities for φ_1 and φ_2 on $\Pr_4$ exceed those on $\Pr_5$, a measure satisfying (**CMF**) must necessarily assign *different* degrees of coherence to the pair of propositions in both situations. Therefore, there can be no measure satisfying both (**CMF**) *and* (**CDI**). Given that (**CDI**) and (**CDI**$_r$) only differ with respect to sets involving at least three propositions, the above example also constitutes a counterexample against the compatibility of (**CDI**$_r$) and (**CMF**).

E. Proof of Theorem 1.9

THEOREM 1.9 (**CLI**$_s$) *entails* (**CLI**$_w$), *but not vice versa.*

PROOF. Let $\mathcal{C}$ be a measure that violates (**CLI**$_w$). Then there is a strongly inconsistent set Δ such that $\mathcal{C}(\Delta) > \min$. Since all strongly inconsistent sets are also weakly inconsistent, $\mathcal{C}$ violates ($\mathbf{CLI}_s$) likewise. On the other hand, recall measure $\mathcal{C}_3$ above. In order to quantify the degree of coherence of a set Γ, $\mathcal{C}_3$ averages the mutual support of all pairs of elements of Γ. Accordingly, assuming that Γ is only weakly independent, $\mathcal{C}_3(\Gamma) > \min$. Thus, $\mathcal{C}_3$ violates ($\mathbf{CDI}_s$) while obviously satisfying ($\mathbf{CDI}_w$). Accordingly, ($\mathbf{CDI}_w$) does *not* entail ($\mathbf{CDI}_s$).

F. Proof of Theorem 1.10

THEOREM 1.10 (**CLE**) *and* (**CSE**) *are logically inconsistent.*

PROOF. Let φ_1 and φ_2 be logically equivalent and consider the following probability distributions:

φ_1	φ_2	Pr_6	Pr_7
T	T	1/8	3/4
T	F	0	0
F	T	0	0
F	F	7/8	1/4

Given the logical equivalence, φ_1 and φ_2 are assigned the maximum degree of coherence $\mathcal{C}$ on each measure satisfying (**CLE**). Particularly, $\mathcal{C}_{\mathrm{Pr}_6}(\varphi_1, \varphi_2) = \mathcal{C}_{\mathrm{Pr}_7}(\varphi_1, \varphi_2)$ for each such measure. On the other hand, given that a measure satisfying (**CSE**) is assumed to assign different degrees of coherence to pairs of equivalent propositions subject to the propositions prior probabilities, each measure $\mathcal{C}'$ satisfying (**CSE**) will assign *different* degrees of coherence to φ_1 and φ_2 on both distributions. Hence, there can be no measure satisfying *both* (**CLE**) and (**CSE**).

G. Proof of Theorem 1.11

THEOREM 1.11 (**CLE**) *and* ($\mathbf{CLI}_i$) *are logically independent for* $i \in \{w, s\}$.

PROOF. We distinguish the following cases:

($\mathbf{CLI}_w$) To show that (**CLE**) and ($\mathbf{CLI}_w$) are logically independent we have to prove the consistency and non-redundancy of these constraints.

(i) *Consistency* The consistency has already been established by the fact that $\mathcal{O}^*$ satisfies both constraints.

(ii) *Non-redundancy* Now we have to show that there are measures that satisfy one of the constraints while violating the other. On the one hand, a measure that satisfies (**CLE**) while violating ($\mathbf{CLI}_w$) is also contained in Table 2, viz., $\mathcal{C}_s$. On the other hand, the refined deviation measure $\mathcal{D}^*$ satisfies ($\mathbf{CLI}_w$) while violating the equivalence constraint (**CLE**). All in all, this establishes the conclusion that those constraints are logically independent.

($\mathbf{CLI}_s$) To show that (**CLE**) and ($\mathbf{CLI}_s$) are logically independent we have, again, to prove consistency and non-redundancy.

(i) *Consistency* The consistency has already been established by the fact that $\mathcal{O}$ satisfies both constraints.

(ii) *Non-redundancy* Remeber that $\mathcal{D}$ satisfies ($\mathbf{CLI}_s$) and violates (**CLE**) while $\mathcal{C}_s$ violates ($\mathbf{CLI}_s$) but satisfies (**CLE**). Accordingly, non-redundancy is already established by Table 2.

H. Proof of Theorem 1.12

Theorem 1.12 (**CSE**) *and* ($\mathbf{CLI}_i$) *are logically independent for* $i \in \{w, s\}$.

Proof. We distinguish the following cases:

($\mathbf{CLI}_w$) To show that (**CSE**) and ($\mathbf{CLI}_w$) are logically independent we have to prove the consistency and non-redundancy of these constraints.

(i) *Consistency* The consistency has already been established by the fact that $\mathcal{D}$ satisfies both constraints.

(ii) *Non-redundancy* $\mathcal{O}$ satisfies ($\mathbf{CLI}_w$) while violating (**CSE**). On the other hand, consider the following measure $\mathcal{C}_4$:

$$\mathcal{C}_4(\varphi_1, \ldots, \varphi_n) = (\Pr(\varphi_1))^{-1}$$

This measure is easily seen to satisfy (**CSE**) while violating ($\mathbf{CLI}_w$). Hence, these latter constraints are logically independent.

($\mathbf{CLI}_s$) As usual, we establish logical independence by proving consistency and non-redundancy:

(i) *Consistency* The consistency has already been established by the fact that $\mathcal{D}$ satisfies both constraints.

(ii) *Non-redundancy* $\mathcal{O}$ satisfies ($\mathbf{CLI}_s$) while violating (**CSE**). Furthermore, $\mathcal{C}_4$ violates ($\mathbf{CLI}_s$) but satisfies (**CSE**). Thus, these constraints are logically independent.

I. Proof of Theorem 1.13

Theorem 1.13 (**Ent**) *and* ($\mathbf{Ent}_c$) *are logically independent.*

Proof. To show that (**Ent**) and (**Ent**$_c$) are logically independent we have to prove the consistency and non-redundancy of these constraints.

(i) *Consistency* The consistency has already been established by the fact that $\mathcal{D}$ satisfies both constraints.

(ii) *Non-redundancy* On the one hand, the overlap measure $\mathcal{O}$ satisfies (**Ent**$_c$) but violates the qualitative constraint (**Ent**). On the other hand, consider the following measure $\mathcal{C}_5$:

$$\mathcal{C}_5(\varphi, \psi) = \Pr(\varphi \wedge \psi) - \Pr(\varphi \wedge \neg\psi)$$

If $\varphi \vDash \psi$, then $\Pr(\varphi \wedge \neg\psi) = 0$ and $\mathcal{C}_5(\varphi, \psi) = \Pr(\varphi) > 0$. Accordingly, assuming that 0 is a plausible threshold for separating coherence and incoherence for $\mathcal{C}_5$, we conclude that this measure satisfies (**Ent**). On the other hand, if φ entails both ψ and ψ', then $\mathcal{C}_5(\varphi, \psi) = \mathcal{C}_5(\varphi, \psi') = \Pr(\varphi)$ irrespective of the posterior probabilities $\Pr(\varphi|\psi)$ and $\Pr(\varphi|\psi')$. Hence, $\mathcal{C}_5$ violates (**Ent**$_c$).

J. Proof of Theorem 1.14

Theorem 1.14 (**Ent**$_c$) *and* (**Dis**$_\vDash$) *are logically equivalent.*

Proof. If $\varphi \vDash \psi$ and $\varphi \vDash \psi'$, then $\Pr(\varphi|\psi) > \Pr(\varphi|\psi')$ iff $\Pr(\psi) = \Pr(\varphi \wedge \psi) + \Pr(\neg\varphi \wedge \psi) < \Pr(\varphi \wedge \psi') + \Pr(\neg\varphi \wedge \psi') = \Pr(\psi')$. Given the logical entailment we conclude that (i) $\Pr(\psi) < \Pr(\psi')$ iff $\Pr(\neg\varphi \wedge \psi) < \Pr(\neg\varphi \wedge \psi')$ and (ii) $\Pr(\varphi \wedge \neg\psi) = \Pr(\varphi \wedge \neg\psi') = 0$. Therefore, we conclude that $\Pr(\psi) < \Pr(\psi')$ iff $\Pr(\varphi \wedge \neg\psi) + \Pr(\neg\varphi \wedge \psi) < \Pr(\varphi \wedge \neg\psi') + \Pr(\neg\varphi \wedge \psi')$. Given that the latter sums are identical to the degree of disagreement between φ and ψ on the one hand, and φ and ψ' on the other, we finally get: if $\varphi \models \psi$ and $\varphi \models \psi'$, then $\Pr(\varphi|\psi) > \Pr(\varphi|\psi')$ iff $dis(\varphi, \psi) < dis(\varphi, \psi')$. Therefore, (**Ent**$_c$) and (**Dis**$_\vDash$) are logically equivalent.

K. Proof of Theorem 1.15

Theorem 1.15 (**Dis**) *entails* (**Dis**$_\vDash$), *but not vice versa.*

Proof. Let $\mathcal{C}$ be a measure violating (**Dis**$_\vDash$), then there are propositions φ, ψ, ψ' such that $\varphi \vDash \psi$, $\varphi \vDash \psi'$, $dis(\varphi, \psi) < dis(\varphi, \psi')$ but $\mathcal{C}(\varphi, \psi) < \mathcal{C}(\varphi, \psi')$. Now, if φ entails both ψ and ψ', then $comp(\varphi, \psi) = \Pr(\varphi \wedge \psi) = \Pr(\varphi) = \Pr(\varphi \wedge \psi') = comp(\varphi, \psi')$. Hence, $\mathcal{C}$ necessarily violates (**Dis**). On the other hand, Table 2 contains numerous measures satisfying (**Dis**$_\vDash$) while violating (**Dis**) so that (**Dis**$_\vDash$) cannot entail (**Dis**).

L. Proof of Theorem 1.16

THEOREM 1.16 (**Ent**) *and* (**Dis**) *are logically inconsistent.*

PROOF. Let $\mathcal{C}$ be a measure satisfying (**Dis**), then $\mathcal{C}(\varphi,\psi)$ has to be monotonically increasing in $comp(\varphi,\psi) = \Pr(\varphi\wedge\psi)$ and monotonically decreasing in $dis(\varphi,\psi)$. Thus, even if $\varphi \vDash \psi$ such that $\Pr(\varphi\wedge\neg\psi) = 0$, we can make $dis(\varphi,\psi)$ as large as we want, for example, by taking $\Pr(\varphi\wedge\psi) = \varepsilon_1$, $\Pr(\neg\varphi\wedge\neg\psi) = \varepsilon_2$ and $\Pr(\neg\varphi\wedge\psi) = 1-\varepsilon_1-\varepsilon_2$. It seems that φ and ψ should turn out incoherent on such a distribution so that $\mathcal{C}$ violates (**Ent**).

M. Proofs for Table 2

(**CDI**)–(**CLI**$_w$) Most of the proofs are given in Schippers [50]. The missing proofs are straightforward and left to the reader.

(**CLI**$_s$) Straightforward in the light of the results on **(CLI**$_w$**)** and 1.9.

(**CSE**) Let $\Delta = \{\varphi_1,\ldots,\varphi_n\}$ be a set of logically equivalent propositions, then we get the following results for those measures violating (**CLE**):

(i) $\mathcal{D}(\Delta) = \Pr(\varphi_i)^{1-n}$

(ii) $\mathcal{D}^*(\Delta) = \frac{1}{n-1}\sum_{k=2}^{n}\sum_{\Delta_i\in[\Delta]^k}\frac{\log_{10}\Pr(\varphi_i)^{1-i}}{m_k}$

(iii) $\mathcal{C}_d(\Delta) = \sum_{(\Delta',\Delta'')\in[\Delta]} l^{-1}\cdot(1-\Pr(\Delta'')) = (1-\Pr(\varphi_i))$

(iv) $\mathcal{C}_r(\Delta) = \sum_{(\Delta',\Delta'')\in[\Delta]} l^{-1}\cdot\Pr(\Delta'')^{-1} = \Pr(\varphi_i)^{-1}$

(v) $\mathcal{C}_l(\Delta) = \sum_{(\Delta',\Delta'')\in[\Delta]} l^{-1}\cdot\Pr(\Delta''|\neg\Delta')^{-1} = \Pr(\varphi_i|\neg\varphi_i)^{-1}$

Thus, $\mathcal{D},\mathcal{D}^*,\mathcal{C}_d$ and $\mathcal{C}_r$ are obviously strictly monotonically decreasing in $\Pr(\varphi_i)$; on the other hand, $\mathcal{C}_l$ is not defined for sets of logically equivalent propositions.

(**Ent**)If φ logically entails ψ, then $\Pr(\psi|\varphi) > \Pr(\psi)$ and therefore all coherence measures based on the incremental concept of confirmation assign a positive degree of coherence to $\{\varphi,\psi\}$. Similarly, $\mathcal{D}(\varphi,\psi) = \Pr(\psi)^{-1} > 1$ and hence $\mathcal{D}^*(\varphi,\psi) > 0$. On the other hand, $\mathcal{O}(\varphi,\psi) = \mathcal{O}^*(\varphi,\psi) = \frac{\Pr(\varphi)}{\Pr(\psi)}$; accordingly, if $\Pr(\varphi)\approx 0$ and $\Pr(\psi)\approx 1$, then $\mathcal{O}(\varphi,\psi) = \mathcal{O}^*(\varphi,\psi) \approx 0$. Therefore, no matter which threshold is chosen, both overlap-based measures will assess $\{\varphi,\psi\}$ incoherent in this case, even though $\varphi \vDash \psi$.

(**Ent**$_c$) If $\varphi \vDash \psi\wedge\psi'$, then $\Pr(\varphi|\psi) > \Pr(\varphi|\psi')$ if and only of $\Pr(\psi) < \Pr(\psi')$.

(i) $\mathcal{D}(\varphi,\psi) = \Pr(\psi)^{-1} > \Pr(\psi')^{-1} = \mathcal{D}(\varphi,\psi')$. An analogous argument shows that $\mathcal{D}^*$ satisfies ($\mathbf{Ent}_c$), too.

(ii) $\mathcal{O}$ can be rewritten as follows:

$$\mathcal{O}(\varphi,\psi) = \left[\Pr(\varphi|\psi)^{-1} + \Pr(\psi|\varphi)^{-1} - 1\right]^{-1}$$

Hence, if $\varphi \vDash \psi \wedge \psi'$, then $\mathcal{O}(\varphi,\psi) > \mathcal{O}(\varphi,\psi')$ if and only if $\Pr(\varphi|\psi) > \Pr(\varphi|\psi')$. Accordingly, both $\mathcal{O}$ and $\mathcal{O}^*$ satisfy ($\mathbf{Ent}_c$).

(iii) A well-known adequacy constraint for probabilistic confirmation measures reads as follows (cf. [32]):

(C) If $\Pr(\varphi|\psi) > \Pr(\varphi|\psi')$, then $\mathfrak{s}(\varphi,\psi) > \mathfrak{s}(\varphi,\psi')$.

Among those measures satisfying this constraint are d, r, l, k and z (cf. [10]). Simple arithmetic manipulations yield the following results for $\varphi \vDash \psi$:

- $\mathcal{C}_d(\varphi,\psi) = 1/2 \cdot (d(\varphi,\psi) + d(\psi,\varphi))$, where $d(\psi,\varphi) = 1 - \Pr(\psi)$. Accordingly, given that d satisfies (C), we infer that $\mathcal{C}_d$ satisfies ($\mathbf{Ent}_c$).
- $\mathcal{C}_r(\varphi,\psi) = 1/2 \cdot (r(\varphi,\psi) + (\Pr(\psi))^{-1})$. An argument like before entails the desired conclusion.
- Although s does not satisfy (C) in general, it is a straightforward task to show that it does when φ logically entails ψ; accordingly, $s(\varphi,\psi) > s(\varphi,\psi')$; furthermore, simple arithmetic manipulations show that $s(\psi,\varphi) > s(\psi',\varphi)$ if and only if $\Pr(\psi) < \Pr(\psi')$. Accordingly, $\mathcal{C}_s$ satisfies ($\mathbf{Ent}_c$).
- Given that $\mathcal{C}_s(\varphi,\psi) = \mathcal{C}_n(\varphi,\psi)$, the latter satisfies ($\mathbf{Ent}_c$), too.
- $\mathcal{C}_l(\varphi,\psi) = 1/2 \cdot \left(\frac{\Pr(\varphi|\psi)}{\Pr(\varphi|\neg\psi)} + \frac{\Pr(\psi|\varphi)}{\Pr(\psi|\neg\varphi)}\right)$, which is again undefined.
- Given that k satisfies (C) we know that $k(\varphi,\psi) > k(\varphi,\psi')$; furthermore, by the fact that φ entails both ψ and ψ' we know that $k(\psi,\varphi) = k(\psi',\varphi) = 1$. Therefore, $\mathcal{C}_k$ satisfies ($\mathbf{Ent}_c$).
- $\mathcal{C}_z$ Analogous to $\mathcal{C}_k$.
- $\mathcal{C}_f$ Analogous to $\mathcal{O}$.

(**Dis**) To show that the overlap measure $\mathcal{O}$ satisfies (**Dis**) note that the measure can equivalently be written as follows:

$$\mathcal{O}(\varphi,\psi) = \frac{comp(\varphi,\psi)}{comp(\varphi,\psi) + dis(\varphi,\psi)}$$

This immediately entails the desired conclusion for $\mathcal{O}$. To see that neither $\mathcal{D}$, $\mathcal{D}^*$ nor any of the mutual support measures satisfies (Dis), consider the following probability distribution, where $\varepsilon = 1 - \Pr(\varphi \vee \psi \vee \varphi' \vee \psi')$.

φ	ψ	φ'	ψ'	Pr	φ	ψ	φ'	ψ'	Pr
T	T	T	T	1/98	F	T	T	T	3/43
T	T	T	F	2/29	F	T	T	F	11/67
T	T	F	T	1/42	F	T	F	T	1/345
T	T	F	F	1292/56115	F	T	F	F	1/21
T	F	T	T	1/45	F	F	T	T	1/42
T	F	T	F	1/8	F	F	T	F	8/59
T	F	F	T	1/100	F	F	F	T	1/141
T	F	F	F	2/27	F	F	F	F	ε

Given this probability distribution we get the following results: $\Pr(\varphi|\psi) \approx 0.31 < 0.36 \approx \Pr(\varphi)$ and $\Pr(\varphi'|\psi') \approx 0.74 > 0.62 \approx \Pr(\varphi')$. Accordingly, each of the incremental confirmation based measures and $\mathcal{D}$ and its refined version agree in that $\{\varphi', \psi'\}$ is more coherent than $\{\varphi, \psi\}$. On the other hand, $comp(\varphi, \psi) = comp(\varphi', \psi')$ and $dis(\varphi, \psi) \approx 0.52 < 0.54 \approx dis(\varphi', \psi')$. Consequently, these measures violate (**Dis**). Furthermore, $\mathcal{C}_f(\varphi, \psi) \approx -0.34 < -0.05 \approx \mathcal{C}_f(\varphi', \psi')$.

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